

Spontaneous Collapse Models on a Lattice

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We present spontaneous collapse models of field theories on a $1 + 1$ null lattice, in which the causal structure of the lattice plays a central role. Issues such as “locality,” “nonlocality,” and superluminal signaling are addressed in the context of the models which have the virtue of extreme simplicity. The formalism of the models is related to that of the consistent histories approach to quantum mechanics.

KEY WORDS: Lattice; collapse models; nonlocality.

1. INTRODUCTION

The spontaneous collapse models of Ghirardi, Rimini, and Weber (GRW), Pearle and others (see ref. 1 for a review) represent a promising direction of research towards an observer independent theory of fundamental matter. These models were first proposed in a nonrelativistic framework and since then much attention has focused on the search for appropriately relativistic models. This is not only important in its own right, but seems to be a prerequisite to any hope of applying collapse model ideas to quantum gravity.

We present a simple collapse model for a field theory defined on a $1 + 1$ null lattice. It is inspired by work by Samols⁽²⁾ and by the interpretation of the GRW model,⁽³⁾ due to Bell,⁽⁴⁾ in which it is the collapse centres that are the “beables” or “real variables.” In the cited work, Bell proved a result suggestive that Lorentz invariant collapse models can be formulated.

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When the system of particles treated by the GRW model can be split into two noninteracting subsystems, the time evolution of one subsystem has no effect on the other, as in standard quantum mechanics. The work presented here can be considered as further support for Bell's expectation that a fully Lorentz invariant collapse model with the ontology he proposed can be constructed. In particular we prove, in the framework of the lattice collapse model, the analogue of Bell's result that, "Events in one system, considered separately, allow no inference [...] about external fields at work in the other."

Although the lattice collapse model is not itself Lorentz invariant, it is not unreasonable to hope that Lorentz invariance will be attained in an appropriate continuum limit. Moreover it is the view of some workers that the aspect of spacetime that is fundamental and survives its encounter with its quantum nemesis is its causal structure and, further, that this fundamental causal structure is discrete.⁽⁵⁾ If one is looking for a development of quantum theory suited to such beliefs about quantum gravity, our model has many attractive features: it is discrete, there is a causal structure, and there is a local evolution rule tied to that causal structure.

2. 1 + 1 LATTICE QUANTUM FIELD THEORY

We briefly review the basics of light cone lattice field theory in 1 + 1 dimensions, introduced in the study of integrable models.⁽⁶⁾ We follow the presentation of Samols⁽²⁾ of this "bare bones" local quantum field theory. Spacetime is a 1 + 1 null lattice, periodically identified in space of width $2N$. We label the links of the lattice L or R depending on whether they are left or right moving null rays. A spacelike surface, σ , is given by the set of links cut by the surface and is specified completely by the position of an initial link and a sequence of N R 's and N L 's labeling the links it cuts successively, moving from left to right, starting with the initial link. An example is shown in Fig. 1, taken from Samols' paper.

The local field variables, α , live on the links. At link l the variable α_l takes just two values, 0 or 1, and there is a ("qubit") Hilbert space, \mathcal{H}_l , spanned by two states labelled by $\alpha_l = 0$ and $\alpha_l = 1$. At each vertex, v , the local evolution law is given by a 4-dimensional unitary R -matrix, $U(v)$, that evolves from the 4-d Hilbert space that is the tensor product of the Hilbert spaces on the two ingoing links to the 4-d Hilbert space on the two outgoing links.

A quantum state $|\Psi\rangle$ on a spacelike surface, σ , is an element of the Hilbert space, \mathcal{H}_σ , that is the tensor product of the Hilbert spaces on all the links cut by σ . $|\Psi\rangle$ is specified in the α basis as a normalised complex function of the variables on the links cut by a spacelike surface, σ . Denoting

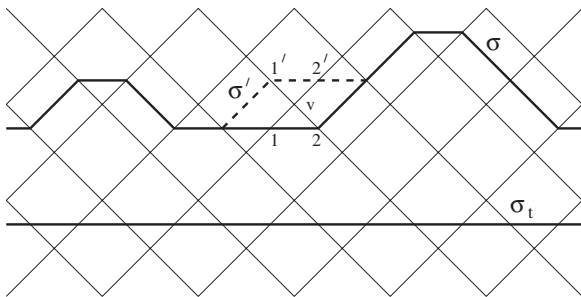


Fig. 1. The light-cone lattice. σ_t is a constant time slice; σ is a general spacelike surface and σ' one obtained from it by an elementary motion across the vertex v .

this set of variables by $\alpha|_\sigma$, the wave function is written $\Psi(\alpha|_\sigma)$. The unitary evolution of the wavefunction to another spacelike surface σ' is effected by applying all the R-matrices at the vertices between σ and σ' , in an order respecting the causal order of the vertices. In the simplest case, when only a single vertex is crossed (to the future of σ) the deformation of σ to σ' is called an “elementary motion” and an example is shown in Fig. 1.

The R-matrices have been left unspecified to keep the discussion as general as possible. In a conventional field theory, they will be uniform over the lattice. One particular choice and a suitable continuum limit leads, for example, to the massive Thirring model.⁽⁶⁾

The standard interpretation of the theory is expressed in terms of the results of measurements of any hermitian operator associated with any surface σ . The state on σ provides the appropriate probability distribution. This standard theory suffers from at least two problems. Firstly, it cannot be a theory of a closed system (the entire universe, say) since it requires external, classical measuring agents. Secondly, serious threats of superluminal signaling arise on trying to extend the interpretation to sequences of measurements tied to spacetime regions more general than hypersurfaces (see, e.g., refs. 7–9) There are strong motivations for trying to develop the field theory into a realistic model in which predictions would be observer independent and superluminal signaling does not occur. We will describe our attempt in the next section but first, for illumination and comparison, we give a brief description of another such model, the Samolsian dynamics.

The Samols model is a realistic, stochastic model of the above lattice quantum field theory that agrees with the predictions of the standard theory in situations where the latter makes predictions. The dynamics is defined inductively. The initial conditions are that on some spacelike surface, σ_0 , the wavefunction is $\Psi(\alpha|_{\sigma_0})$ and a configuration $\hat{\alpha}|_{\sigma_0}$ is chosen at random according to the quantum mechanical probability distribution $|\Psi(\hat{\alpha}|_{\sigma_0})|^2$.

Suppose we have a surface σ_{k-1} with wavefunction and some realised field configuration on it. One of the possible elementary motions occurs thus: at random one of the RL pairs is chosen from the sequence of links that defines the surface σ_{k-1} and the surface is moved up across the the associated vertex so that the pair is replaced by LR. As in Fig. 1, let this motion be from links 1 and 2 to 1' and 2'. The wavefunction is evolved forward to σ_k by the R-matrix of the vertex. Field values are realised randomly on the new links according to the conditional probability, $f_{\Psi}(\hat{\alpha}|\sigma_{k-1} \rightarrow \sigma_k)$, of realising values $(\hat{\alpha}_{1'}, \hat{\alpha}_{2'})$ given all the realised values $\hat{\alpha}_l$ up to then, where

$$f_{\Psi}(\hat{\alpha}|\sigma_{k-1} \rightarrow \sigma_k) = \frac{|\Psi(\alpha|_{\sigma_k})|^2}{\sum_{\alpha_{1'}, \alpha_{2'}} |\Psi(\alpha|_{\sigma_k})|^2} \Big|_{\alpha|_{\sigma_k} = \hat{\alpha}|_{\sigma_k}} \quad (2.1)$$

This rule guarantees that the marginal probability distribution on $\alpha|_{\sigma_k}$ is equal to the quantum mechanical one. It also means that there is no conditional dependence of $(\hat{\alpha}_{1'}, \hat{\alpha}_{2'})$ on the realisations to the past of σ_k .

For each sequence of hypersurfaces generated by possible successive elementary motions, $\gamma = \{\sigma_0, \sigma_1, \sigma_2, \dots\}$, this gives a probability distribution on the sample space of all field configurations (on and) to the future of σ_0 . To get the unconditional probability distribution we sum these over γ with weights given by the stochastic rule for elementary motions stated above.

The essential structure of both the basic lattice quantum field theory and the Samolsian dynamics is very simple and versatile. It requires only a discrete causal structure and a local unitary evolution, so the generalisation can easily made to the case of a quantum field theory on any locally finite partial order (a ‘‘causal set’’⁽⁵⁾) where the field variables live on the links, as Samols describes.

3. GRW ON THE LATTICE

One of the defining features of Samolsian dynamics is that it is operationally equivalent to standard quantum theory in situations where the standard theory applies. Someone who believes that standard quantum theory will never be found to give incorrect predictions may consider this essential, but for those of us who keep a more open, scientific mind it is interesting to consider alternatives that give rise to predictions that differ from those of the standard theory. Spontaneous collapse models are such alternatives and so let us now construct a collapse version of the lattice field theory.

In the original GRW dynamics, the wave function is a function of the position of the particle. When a collapse happens it is centred on a particular,

randomly chosen position and according to the Bell interpretation, that position at that time—that *event*—is then real. We are considering a field theory here and the quantum state is a functional of the field configuration on a spacelike surface. So, now, collapses will be centred on field values and it will be one, randomly chosen field configuration on the lattice that will constitute reality in our model, which proceeds inductively as follows.

We start with a wave function $\Psi(\alpha|_{\sigma_0})$ on a spacelike surface σ_0 .

1. Suppose we have $\Psi(\alpha|_{\sigma_{k-1}})$ on surface σ_{k-1} . At random, an elementary motion occurs and the wavefunction Ψ is evolved forward by the unitary R-matrix associated with the single vertex, v_k , crossed, to the new surface σ_k . There the resulting wavefunction is $\Psi(\alpha|_{\sigma_k})$.
2. A field value $\hat{\alpha}_L$ is realised on the new L link. The value is chosen at random from $\{0, 1\}$ according to the GRW probability distribution $(N_L(\hat{\alpha}_L))^2$ which will be defined shortly.
3. The wave function on the surface σ_k suffers a “hit” and becomes

$$\Psi'(\alpha|_{\sigma_k}) = \frac{j_{\alpha_L \hat{\alpha}_L} \Psi(\alpha|_{\sigma_k})}{N_L(\hat{\alpha}_L)} \tag{3.1}$$

where the GRW “jump factor” is given by

$$j_{\alpha_L \hat{\alpha}_L} = \frac{\delta_{\alpha_L \hat{\alpha}_L} + (1 - \delta_{\alpha_L \hat{\alpha}_L}) X}{\sqrt{1 + X^2}} \tag{3.2}$$

with $0 \leq X \leq 1$ and the normalisation given by

$$(N_L(\hat{\alpha}_L))^2 = \sum_{\alpha|_{\sigma_k}} j_{\alpha_L \hat{\alpha}_L}^2 |\Psi(\alpha|_{\sigma_k})|^2 \tag{3.3}$$

which is the probability distribution in step 2.

Just a word of explanation so that the notation is clear. The link, L , is one of the links in σ_k and in Eq. (3.1) α_L is therefore one of the field variables in the argument of the wavefunction. For field configurations on σ_k in which the variable α_L agrees with the value $\hat{\alpha}_L$ (i.e., the realised value) the amplitude for that field configuration is multiplied by the factor $1/(N_L \sqrt{1 + X^2})$ and otherwise the amplitude is multiplied by $X/(N_L \sqrt{1 + X^2})$. Thus the jump factor j_L is chosen so that the amplitudes of field configurations that agree with the realised value $\hat{\alpha}_L$ are enhanced over the amplitudes of field configurations that do not by the ratio $1/X$. For example, if $\hat{\alpha}_L = 1$ then the effect of the multiplication of the wave function by the

jump factor is to act on the two dimensional Hilbert space for the link L by the matrix

$$\frac{1}{\sqrt{1+X^2}} \begin{pmatrix} 1 & 0 \\ 0 & X \end{pmatrix} \quad (3.4)$$

where the first (last) row is labeled by the state with $\alpha_L = 1$ ($\alpha_L = 0$).

4. A collapse occurs on the new right link, R , to a field value $\hat{\alpha}_R$. The value is chosen at random from $\{0, 1\}$ according to the probability distribution $(N_R(\hat{\alpha}_R))^2$.

5. The wave function on the surface σ_k suffers a second ‘hit’ and becomes

$$\Psi''(\alpha|_{\sigma_k}) = \frac{j_{\alpha_R \hat{\alpha}_R} \Psi'(\alpha|_{\sigma_k})}{N_R(\hat{\alpha}_R)} \quad (3.5)$$

where

$$j_{\alpha_R \hat{\alpha}_R} = \frac{\delta_{\alpha_R \hat{\alpha}_R} + (1 - \delta_{\alpha_R \hat{\alpha}_R}) X}{\sqrt{1+X^2}} \quad (3.6)$$

and

$$(N_R(\hat{\alpha}_R))^2 = \sum_{\alpha|_{\sigma_k}} j_{\alpha_R \hat{\alpha}_R}^2 |\Psi'(\alpha|_{\sigma_k})|^2 \quad (3.7)$$

which is the probability distribution for step 4.

6. Go to step 1 where now it is the wavefunction Ψ'' that is evolved forward by the R -matrix of the next randomly chosen vertex.

It will be convenient in what follows to refer to the realisation of α values on the links R and L , outgoing from v_k , as a single *event at the vertex*, v_k . The values $\{\alpha_R, \alpha_L\}$ are summarised as α_{v_k} . The dynamics can then be re-expressed as an elementary motion followed by a single realisation of value $\hat{\alpha}_{v_k}$ with jump factor

$$j_{\alpha_{v_k} \hat{\alpha}_{v_k}} \equiv j_{\alpha_L \hat{\alpha}_L} j_{\alpha_R \hat{\alpha}_R} \quad (3.8)$$

and probability distribution

$$(N(\hat{\alpha}_{v_k}))^2 = \sum_{\alpha|_{\sigma_k}} j_{\alpha_{v_k} \hat{\alpha}_{v_k}}^2 |\Psi(\alpha|_{\sigma_k})|^2. \quad (3.9)$$

A given “run” of the dynamics will generate a random sequence, $\gamma = \{\sigma_1, \sigma_2, \dots\}$, of surfaces to the future of the initial surface, σ_0 , each related to the previous one by an elementary motion. A sequence γ is equivalent to a linear ordering of the vertices to the future of the initial surface that is compatible with the causal order of the vertices (called a “linear extension” of the causal order or a “natural labeling” of the vertices).

The probability distribution generated by this dynamics is, as in the Samolsian dynamics, a measure on the sample space, Ω , of all possible field configurations to the future of σ_0 . Strictly, what the dynamics gives is a probability measure on certain subsets of Ω , the so-called “cylinder sets.” A cylinder set consists of all field configurations that agree with a given one on a “partial stem” which is a finite set of vertices that contains its own causal past (to the future of σ_0). Standard measure theory then guarantees that this extends to a measure on the σ -algebra generated by the cylinder sets, that is all sets formed by countable set operations on the cylinder sets.

In the Samolsian dynamics, the probability distributions that are conditioned on γ are not equal. To obtain the full distribution, these are summed over all γ (all natural labelings of the vertices). Also, although the probability distribution on the possible events at a vertex (i.e., field values on a single outgoing LR pair) is independent of the R-matrices spacelike to it, this is not true of the probability distribution on the events in a more general spacetime region.

By contrast, and as anticipated by Samols, the enhanced locality property of our collapse model means that the marginal probability distribution on any collection of events is independent of the R-matrices at vertices spacelike to the whole collection. Moreover, the probability distributions conditioned on γ are equal to each other. These claims are proved in the next section. This means that the order of evolution of the surfaces can be considered to be genuinely without physical meaning (in contrast to the Samolsian dynamics within which the sequence of hypersurfaces is without *operational* meaning because a local observer cannot determine it). We can still, if we wish, consider the full probability distribution to be given by a sum over γ of the distributions conditioned on γ but the contribution from each is the same. This is the analogue, in this setting, of general covariance: the independence of the action of an unphysical labeling.

4. THE PROBABILITY DISTRIBUTION IS INDEPENDENT OF THE SEQUENCE OF SURFACES

We show that the probability distribution is independent of the sequence of hypersurfaces. Consider a sequence of surfaces, specified by a

natural labeling of the vertices to the future of σ_0 , $\{v_1, v_2, v_3, \dots\}$. Let the surface in the sequence just after the elementary motion across v_k be denoted σ_k and the two outgoing links from v_k be L_k and R_k . The variable α_{v_k} stands for $\{\alpha_{L_k}, \alpha_{R_k}\}$. Let the Hilbert space on σ_k be denoted \mathcal{H}_{σ_k} and recall that it is the tensor product of $2N$ 2-d Hilbert spaces, one for each link cut by σ_k . The state on σ_k , just after the evolution by the R-matrix at v_{k-1} but before the hit, will be denoted by $|\Psi_k\rangle$, and the state after the hit will be denoted by $|\Psi'_k\rangle$. $|\Psi_k\rangle$ depends on the realised values $\{\hat{\alpha}_{v_1}, \dots, \hat{\alpha}_{v_{k-1}}\}$ and $|\Psi'_k\rangle$ depends on $\{\hat{\alpha}_{v_1}, \dots, \hat{\alpha}_{v_k}\}$. They are related by

$$|\Psi'_k\rangle = \frac{J(\hat{\alpha}_{v_k}) |\Psi_k\rangle}{N_k(\hat{\alpha}_{v_k})} \quad (4.1)$$

where $J(\hat{\alpha}_{v_k})$ is the linear operator defined as follows. $J(\hat{\alpha}_{v_k})$ acts on the four-dimensional Hilbert space associated with the outgoing links from v_k as the matrix:

$$J(\hat{\alpha}_{v_k})_{\alpha_{L_k} \alpha_{R_k} \beta_{L_k} \beta_{R_k}} = j_{\alpha_{L_k} \hat{\alpha}_{L_k}} \delta_{\alpha_{L_k} \beta_{L_k}} j_{\alpha_{R_k} \hat{\alpha}_{R_k}} \delta_{\alpha_{R_k} \beta_{R_k}} \quad (\text{no sums}). \quad (4.2)$$

with $j_{\alpha_{L_k} \hat{\alpha}_{L_k}}$ given by Eq. (3.2). $J(\hat{\alpha}_{v_k})$ acts as the identity on the other Hilbert spaces in the tensor product \mathcal{H}_{σ_k} .

Denote the R-matrix at v_k by $U(v_k)$.

We claim that the probability that the field values $\{\hat{\alpha}_{v_1}, \dots, \hat{\alpha}_{v_n}\}$ are realised, given the sequence of surfaces $\gamma = \{\sigma_1, \sigma_2, \dots\}$, is

$$P^\gamma(\hat{\alpha}_{v_1}, \dots, \hat{\alpha}_{v_n}) = |J(\hat{\alpha}_{v_n}) U(v_n) J(\hat{\alpha}_{v_{n-1}}) U(v_{n-1}) \cdots J(\hat{\alpha}_{v_1}) U(v_1) |\Psi_0\rangle|^2. \quad (4.3)$$

Proof of Claim. By induction. The probability that $\alpha_{v_1} = \hat{\alpha}_{v_1}$ is

$$P^\gamma(\hat{\alpha}_{v_1}) = |J(\hat{\alpha}_{v_1}) U(v_1) |\Psi_0\rangle|^2. \quad (4.4)$$

Assume that Eq. (4.3) is true for $n = k-1$. Then

$$P^\gamma(\hat{\alpha}_{v_1}, \dots, \hat{\alpha}_{v_k}) = P^\gamma(\hat{\alpha}_{v_k} | \hat{\alpha}_{v_1}, \dots, \hat{\alpha}_{v_{k-1}}) P^\gamma(\hat{\alpha}_{v_1}, \dots, \hat{\alpha}_{v_{k-1}}) \quad (4.5)$$

$$= |J(\hat{\alpha}_{v_k}) |\Psi_k\rangle|^2 |J(\hat{\alpha}_{v_{k-1}}) U(v_{k-1}) \cdots U(v_1) |\Psi_0\rangle|^2 \quad (4.6)$$

$$\begin{aligned} &= \frac{|J(\hat{\alpha}_{v_k}) U(v_k) J(\hat{\alpha}_{v_{k-1}}) |\Psi_{k-1}\rangle|^2}{|J(\hat{\alpha}_{v_{k-1}}) |\Psi_{k-1}\rangle|^2} \\ &\quad \times |J(\hat{\alpha}_{v_{k-1}}) U(v_{k-1}) \cdots U(v_1) |\Psi_0\rangle|^2. \end{aligned} \quad (4.7)$$

We may replace $|\Psi_{k-1}\rangle$ in the numerator and denominator of the fraction by $U(v_{k-1})J(\hat{\alpha}_{v_{k-2}})\cdots U(v_1)|\Psi_0\rangle$ as the normalisation factors cancel out and the result follows.

From this we can see that the probability is unchanged by an exchange of the order of any two successive spacelike separated vertices in γ , say v_l and v_{l+1} , because

$$[J(\hat{\alpha}_{v_l})U(v_l), J(\hat{\alpha}_{v_{l+1}})U(v_{l+1})] = 0 \tag{4.8}$$

if v_l and v_{l+1} are spacelike. Any order preserving list, γ , of finitely many vertices can be transformed into any other order preserving list, γ' , of the same vertices by a sequence of such exchanges. Thus, the probability distribution on $\{\hat{\alpha}_{v_1}, \dots, \hat{\alpha}_{v_n}\}$ is independent of γ .

We use this to show that the model satisfies what we call ‘‘external relativistic causality.’’ By this we mean that external agents that exist in spacetime in addition to the field and that can affect the field only by changing the R-matrices locally at their spacetime position cannot use the field to send superluminal signals. Suppose agent Alice is located in spacetime region A and agent Bob in region B such that all vertices of A are spacelike to all vertices of B . Suppose Bob has some records of past events in the causal past of B . Can Alice signal to Bob by manipulating the R-matrices in A ? This can only happen if the probability distribution on a set of events in B , conditional on some collection of events in $P(B)$ (the causal past of B), depends on an R-matrix in A . This probability distribution can be calculated from the joint distribution on all events in B and $P(B)$, $P(\hat{\alpha}_B, \hat{\alpha}_{P(B)})$ where $\hat{\alpha}_B$ is shorthand for the α values in B etc. To calculate these probabilities we may use any natural labeling of the vertices to the future of σ_0 . There exists a natural labeling that first labels the vertices in $P(B)$ and then those in B . Since A intersects neither B nor $P(B)$, we see that $P(\hat{\alpha}_B, \hat{\alpha}_{P(B)})$ is independent of the R-matrices in A .

The further question arises of whether the model satisfies ‘‘internal relativistic causality’’ where this would mean that if the field were the entire universe (with no external agents) no superluminal signaling could occur in that universe. This demands that a definition of ‘‘superluminal signaling’’ be made in this case. Such a definition does not exist, but preliminary versions are being worked on.⁽¹⁰⁾

We may adopt a ‘‘Heisenberg picture’’ description and throw the evolution onto the J 's by defining

$$J_{v_k}(\hat{\alpha}_{v_k}) = U^{-1}(v_1)\cdots U^{-1}(v_k)J(\hat{\alpha}_{v_k})U(v_k)\cdots U(v_1). \tag{4.9}$$

Note that the R-matrices spacelike to v_k can be commuted through the expression and cancelled off with their inverses so that $J_{v_k}(\hat{\alpha}_{v_k})$ only depends on the R-matrices in the causal past of v_k . Then (4.3) becomes

$$P^\gamma(\hat{\alpha}_{v_1}, \dots, \hat{\alpha}_{v_n}) = |J_{v_n}(\hat{\alpha}_{v_n}) \cdots J_{v_1}(\hat{\alpha}_{v_1}) |\Psi_0\rangle|^2. \quad (4.10)$$

We see an immediate similarity with the form of the probability of a history in the consistent (or decoherent) histories approach to quantum mechanics due to Griffiths, Omnès, Gell-Mann, and Hartle. Differences with the consistent histories approach include the fact that the J operators are, in general, not projectors (though they satisfy $\sum_{\hat{\alpha}_{v_k}} J^2(\hat{\alpha}_{v_k}) = 1$ and are examples of what are known as “Kraus operators”) and the histories are not “consistent.”

A translation of the nonrelativistic GRW model into this “historical” framework was made by Kent.⁽¹¹⁾ Connections between consistent histories and collapse models and related theories of quantum open systems have also been made by Diosi, Gisin, Halliwell, and Percival and by Brun.^(12–14) It seems possible, following Kent, following the consistent historians, to adopt the causally ordered list of J operators,

$$\{J_{v_1}(\hat{\alpha}_{v_1}), J_{v_2}(\hat{\alpha}_{v_2}), \dots, J_{v_n}(\hat{\alpha}_{v_n})\} \quad (4.11)$$

as the specification of the history of which Eq. (4.10) is the probability. That is, it appears that this is a possible alternative ontology for our model, different from the field configurations (the J 's depend on the number X for example, whereas the field configurations do not). Whether or not these are genuinely different ontologies, and what that would mean if they nevertheless produce the same predictions, seems a subtle question, beyond the scope of the present paper.

5. DISCUSSION

The main value of our model is that it is very simple and straightforwardly illuminates many of the issues that arise in seeking realistic alternatives to standard quantum mechanics. The model has a high degree of locality built into it and external agents cannot manipulate it to produce superluminal signals. However, there *are* “nonlocal correlations:” the probability distribution on events at a vertex will generally depend on the events realised at vertices spacelike to it. In Rideout and Sorkin’s terminology, the model does not satisfy “Bell causality”⁽¹⁵⁾ and so it has the potential of reproducing the Bell-inequality-violating correlations that most likely occur in nature.

The simplicity of (4.10) and its connection to the consistent histories formalism suggest numerous potential variations: different choices of the jump or Kraus operators, branch dependence (where the choice of J 's at a vertex can depend on events in its causal past), using a mixed state instead of a pure state, and adding a final state.

With a given choice of R-matrices, the continuum limit may be examined. This would be done by a procedure of coarse graining and renormalisation of the X parameter, studying the limit of the probability distribution to see if it tends to something well-defined. With some choices, it is possible that the continuum limit could be one of the models studied by Adler and Brun⁽¹⁶⁾ as these share one of the main features of our lattice model, namely that it is constructed using a locally, randomly evolving surface. The Adler–Brun models display an unphysical large energy production and we can ask whether our lattice model can be adjusted so as to avoid this. For example, one variation of the model would be to let the realisation of a field value on each pair of links have only a probability of occurring which would have the effect of making the field configuration sparser. The probability of realisation at a vertex could be fixed or could vary according to events in the causal past.

Because of its extreme simplicity, the model seems well suited to exploring the issue of “internal” relativistic causality, as mentioned above. That is, we can try to formulate a definition of internal relativistic causality for the model—some condition on the probability distribution on field configurations—and then see if it is satisfied. It may be, however, that this definition will also depend on the relationship of the field configurations to the macroscopic world of experiments and experimenters and will be hard to glean without this having been determined.

This is connected to the major question: what, if anything, do these models describe physically? Do the realised field configurations exhibit interesting behaviour or are they just too noisy to see any structure (note that when the parameter X is chosen to be 1, the evolution of the state is the standard quantum mechanical unitary evolution and the unrelated probability distribution on the field values is that on a set of independent variables which are 0 or 1 with probability $\frac{1}{2}$). Do superpositions of macroscopically distinct configurations—whatever that means in this context—collapse onto one or other of those configurations, as we would want? Simulations are being done to investigate these questions.⁽¹⁷⁾

These simulations may also throw light on the issue, raised by Kent,⁽¹⁸⁾ of the status of the quantum state in collapse models with the Bell interpretation. We wish to say that the real variables are the field values alone. For this to be a satisfying interpretation, it seems necessary, as Kent stresses, that the quantum state on any spacelike surface be equivalent to the

historical record of events to the past of that surface. If it were not so then the information about the quantum state would be needed, over and above the information about past and present events, in order to be able to make predictions about the future. Denying the reality of the quantum state would then be an awkward thing to have to do. The simulations in progress will test the hypothesis that in the limit of late times, the probability distribution on future events depends only on past and present events and not on the quantum state on the initial hypersurface. We could then interpret the quantum state on a given hypersurface as a useful fiction, an executive summary of the past that allows future prediction. If this turns out not to be the case, however, one could still consider the model using the interpretation in which the state is taken to be real.

The null lattice used here is special to $1+1$ dimensions and this presents a difficulty in generalising our model to higher dimensions. However, from the point of view of the causal set approach to quantum gravity, spacetime is a continuum approximation to a discrete underlying substructure (reality) and this substructure is a causal set, which, as mentioned previously, is a locally finite partial order. In the case of a continuum approximation that is $d+1$ dimensional Minkowski spacetime (perhaps identified on a d -torus) then the underlying reality is a causal set that can be produced by sprinkling points into the spacetime by a unit density Poisson process (i.e., the mean number of points in any region is equal to the volume of the region in Planck units) and endowing them with the partial order that they inherit from the spacetime causal order. The rules of a collapse model on a background causal set would involve putting field variables and associated Hilbert spaces on the links and unitary R -matrices at the vertices—as described by Samols⁽²⁾—and supplementing this by placing jump (Kraus) operators on the links also. This seems feasible. If the observer independent successor theory to standard quantum theory turns out to be something along these lines, involving the discrete causal structure that underpins the spacetime continuum approximation, it might then be said that the resolution of the measurement problem does indeed involve gravity as many workers have suggested.

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